Progression in Calculation

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of 'real' life. Games and songs are a useful way to introduce key vocabulary involved in the four mathematical operations.



- > Children should be encouraged to read number sentences aloud in different ways. 3 + 2 = 5
 - "Three add two equals 5", "5 is equal to three and two" or
 - " 5 is the same as three and two".
- > Children make a record in pictures, words or symbols of addition activities.





> Solve simple problems using fingers.



Introduce number tracks to count-up on and to find one more: What is one more than 4?



> Use number lines alongside number tracks and practical apparatus to solve addition calculations and word problems.



Children will need the opportunity to look at and talk about different models and images as they move between different representations.

Addition KS 1 & 2

Key language which should be used:

sum, total, parts and wholes, plus, add, score, altogether, more than, 'is equal to', 'is the same as'.



		Concrete	Pictorial	Abstract
	uo bu		12 + 5 = 17	12 + 5 = 17
	oer and counti	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	10 11 12 13 14 15 16 17 18 19 20	Place the larger number in your head and count on the smaller number to find your answer.
Year 1	ger num		Start at the larger number on the number line and count on in ones or in one jump to find the answer.	
	Starting at the big	5 6 7 8 Count on using number lines by using counters, cubes or numicon.	A bar model which encourages the children to count on.	
Year 1	egrouping to make 10	6 + 5 = 11 Use counters and tens frames or Numicon. Start with the bigger number and use the smaller number to make 10. Then add on the rest.	Children to draw the tens frame and counters/cubes.	Children develop an understanding of what is happening with the numbers and do this in their head. 6+5=11 4 1
	2			6 + 4 =10 10 + 1 = 11

	Concrete	Pictorial	Abstract
year 2 Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7. Image: Comparison of the state of the state of the digits (if possible) then add on the third digit.	x x o o o x x o o o o * * * * * * braw the tens frame and counters. Draw the tens frame and counters. Draw the tens frame and counters.	4 + 7 + 6 = 10 + 7 $= 17$ Combine the two numbers that make 10 and then add on the remainder.
Year 2 TO + O Column method- no regrouping	Continue to develop understanding of partitioning and place value. 41 + 8	Children can represent the concrete using a particular symbol e.g. lines for tens and dot/squares/crosses for ones	41 + 8 1 + 8 = 9 40 + 9 = 49 40 40 1 40 40 1 40 40 40 40 40 40 40 40

		Concrete	Pictorial	Abstract
	nethod- Ng	Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 24 + 15=	After practically using the base 10 blocks and place value counters, children can draw the blocks/counters to help them to solve additions. 32 + 23 =	21 + 42 = 20 + 40 = 60 1 + 2 = 3 60 + 3 = 63
Year 2	TO + TO Column m no regroupin			$\frac{\text{Calculations}}{21 + 42} = \frac{21}{42}$
Year 2	TO + TO Column method- with regrouping	Make both numbers on a place value grid. Tens Ones + Image: Construction of the second sec	Children can represent the concrete in one of two ways:	40 + 9 + <u>20 + 3</u> 60 + 12 = 72 Formal method: 49 + <u>23</u> <u>72</u> 1





Subtraction Early Years

Useful guidance, models and images.

Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left. E.g. 5 - 2 =



 Construct number sentences verbally or using cards to go with practical activities.



- Children should be encouraged to read sentences aloud in different ways..."five subtract one leaves four", "four is equal to five subtract one" or " four is the same as five subtract one".
- > Children make a record in pictures, words or symbols of subtraction activities.



- > Solve simple problems using fingers.
- Introduce number tracks to c 1 2 3 4 5 6 o find one less: What is 1 less than 6?

Key language which should be used:

Take (away), estimate, leave, left, fewer, less, difference between, the same as, counting/hopping back. How many are left/left over? How many have gone - one less, two less, three less... How many fewer is...? Number lines can be used alongside number tracks and practical apparatus to solve subtraction calculations and word problems. Children count back showing hops back on the number line.



Children will need the opportunity to look at and talk about different models and images as they move between different representations.

Subtraction KS 1 & 2 Key language which should be used: Take away, less than, the difference, subtract, minus, fewer, decrease, '7 take away 3', 'the difference is four'. Concrete Pictorial Abstract Cross out drawn objects to show what has been taken Use physical objects, counters, cubes etc. to show how objects can be physically taken away. 4 - 3 =away and removed from the whole. 4 - 2 = 2 6 - 2 = 4 = 4 - 3 Encourage children to understand how the abstract idea can be visualised by using numbers in a Taking away ones bar model or part whole model. Use a pictorial representation of objects to show the Link to addition- use the part whole model to help explain the inverse between addition part part whole model. Year 1 4 and subtraction. 3 ? If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =

		Concrete	Pictorial	Abstract
		Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 - 4	Draw a bar and cross off the amount. 4 - 3 = 1 Count back on a number line or number track.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.
Year 1	Counting back	13 4 Use cubes and move them away from the group as you take them away counting backwards as you go. Image: Comparison of the state of the	Count back on a number line or number track. 13 - 4 = 9 9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line. NB: Later this can progress all the way to counting back using two 2 digit numbers: -10 -10 -10 -10 -10 -10 -10 -10 -10 -10 -10 -10 -10 -10 -10 -10 -10 -10 -57	

		Concrete	Pictorial	Abstract
			13 - 7 = 6 $3 4$ $3 4$ $3 4$ $3 4$ $3 4$ $3 4$ $3 4$ $3 6$	14 - 5 = 9 How many do we take off to reach the next 10? How many do we have left to take
Year 1	Aake 10	Make 14 on the ten frame. Take away the four first to make 10 and then takeaway	You have reached your answer.	Children to represent it how they have solved it e.g.
	~	one more so you have taken away 5. You are left with the answer of 9. This can also be done using Numicon.	Children can draw the tens frame.	14 - 5 = 914 is made up of 5, 5 and 4 so I can subtract one 5 to be left with554 and 514 - 5 = 95 is made up of 4 and 1 so I can subtract 4 to make 10 and then 1 to get to 9
Year 1	Find the difference	Compare amounts and objects to find the difference. Use cubes to build towers or make bars to find the difference. Use basic bar models with items to find the difference.	Children can draw cubes/other concrete objects that they have used. XXXXXXXXXXXX XXXXXXXXX They can use a bar model. They can draw a number line and count on to find the difference.	Children should understand how to represent the abstract idea as a written calculation. Comparison Bar Models Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them. 13 22 22 22 22 - 13 = 9







Fluency variation,	different ways to ask child	ren to solve addition calculat	ions e.g. 21 + 34
Fluency variation,	different ways to ask child Raj spent £391, Timmy spent £186. How much more did Raj spend? I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?	ren to solve addition calculat 391 - 186 391 -186 Find the difference between 391 and 186. Subtract 186 from 391. What is 186 less than 391?	ions e.g. 21 + 34 What's the calculation? What's the answer?

Multiplication Early Years

Useful guidance, models and images.

- > The link between addition and multiplication can be introduced through doubling.
- > Numicon can be used to visualise the repeated adding of the same number.



Children can record this by printing or drawing around the Numicon pieces.

> Begin with mostly concrete or pictorial representations.



e.g. How many groups of 2 are there? 2 + 2 + 2 + 2 + 2, so 5 groups of 2

> Use 'real' life contexts and use of practical equipment to count in repeated groups of the same size.





How many wheels are there altogether?

How much money do I have?

> Count in twos, fives, tens both aloud and with objects.



Key language which should be used:

Lots of, groups of, times, multiply, multiplied by, multiple of. Once, twice, three times... ...times as (big, long, wide...) repeated addition double estimate add again and again



Multiplication KS 1 & 2

Key language which should be used:

Double, times, multiplied by, the product of, groups of, lots of, 'is equal to', 'is the same as'



		Concrete	Pictorial	Abstract
		Count in multiples supported by concrete objects in equal groups.	Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud.
	ltiples		Sur sur sur sur sur	Write sequences with multiples of numbers.
Year 1/2	nting in mu		6 <u>5</u> 10 15 20 25 30	2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
	Cou	<u>44</u> -4 -6 <u>111111111111111111111111111111111111</u>		Abstract number line 3 x 4
'ear 1/2	ving commutative iplication	Create arrays using counters/ cubes to show multiplication sentences.	Draw arrays in different rotations to find commutative multiplication sentences.	Use an array to write multiplication sentences and reinforce repeated addition.
~	Arrays- shov mult	4 x 6 = 6 x 4	Link arrays to area of rectangles.	5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 $5 \times 3 = 15$ $3 \times 5 = 15$

		Concrete	Pictorial	Abstract
Year 3	Partition to multiply	Use Base 10, Numicon, Cuisenaire rods) 4 x 15 15 x 4	Children can represent the concrete manipulatives in a picture e.g. Base 10 can be represented like 15 x 4 T 0 XXXXX XXXXX XXXXX XXXXX XXXXX XXXXX XXXXX XXXXX XXXXX	Children to be encouraged to show the steps that they have taken. 4×15 $10 \times 4 = 40$ $5 \times 4 = 20$ 40 + 20 = 60 A number line can also be used.
Year 3/4	Grid Method, no exchanging	Use Base 10 or place value counters. 3 x 23 Make 23, 3 times. See how many ones, then how many tens 10 10 1 1 1 10 10 1 1 1 10 10 1 1 1 10 10 1 1 1 10 10 1 1 1	Children to represent the counters in a pictorial way.	Children to record what it is they are doing to show understanding 3×23 20 3 $3 \times 20 = 60$ $3 \times 3 = 9$ 60 + 9 = 69 23 $\frac{\times 3}{69}$



		Concrete	Pictorial	Abstract
Year 5/6	Expanded method	Show the link with arrays to first introduce the expanded method. 10 8 10 80 3 80 80 24	$\begin{array}{c ccccc} x & 1 & 0 & 8 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 80 \\ \hline 3 & 0 & 30 & 0000000 \\ \hline 3 & 0 & 30 & 24 \end{array}$	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. 18 x <u>13</u> 24 (3 x 8) 30 (3 x 10)) 80 (10 x 8) <u>100</u> (10 x 10) 234
Year 5/6	Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. $\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$

Fluency variation, different	ways to ask children to solve (5 x 23
23 23 23 23 23 23 23 23 23 100	Find the product of 6 and 23 $6 \times 23 =$ eek. ks? 6×23 5×23 6×23 $\times 23 \times 6$ $\times 23 \times 6$ $\times 7$ $\times 7$ $\xrightarrow $	What's the calculation? What's the answer? Image: Comparison of the compari

Division Early Years

Usefu	l guidance, models and images.	Key language which should be used:
\succ	Solve problems including doubling, halving and sharing.	Halve, share, share equally,
\succ	Show children representations of division as grouping and sharing.	one each, two each
\triangleright	Introduce through halving.	group in pairs, threes,
	e.g. Concrete and pictorial representations linked to 'real' life.	equal groups of
		divide
		divided by
		divided into
		left over
\triangleright	Grouping	estimate
	Mum has 6 socks. She grouped them into pairs – how many pairs	fraction
	did she make? How many socks did she have altogether?	half
	111	halves
		whole
		quarter
\triangleright	Sharing - this is a useful way of introducing young children to	
	fractions and calculating with fractions.	
	e.g. I have ten sweets. I want to share them with my friend. How	
	many will we each have?	
	I have got a whole pizza to share between two people. Can you cut	
	the pizza in half?	
\succ	Children can record in pictures, words or symbols of division	
	activities.	

Division KS 1 & 2

Key language which should be used:

Share, group, divide, divided by, half, 'is equal to', 'is the same as'.

	<u> </u>	Concrete	Pictorial	Abstract
	Sharing objects into groups	I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities. 32 32 32 32 32 32 32 32	Share 9 buns between three people. 9 ÷ 3 = 3
Year 1/2	Division as grouping	Understand division as repeated grouping and subtracting. Use cubes, counters, objects etc. to aid understanding. $06 \div 3 = 32$	Use a number line and pictures of counters to show jumps in groups. The number of jumps equals the number of groups.	Abstract number line.

		Concrete	Pictorial	Abstract
Year 1/2	Division within arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created.		Find the inverse of multiplication and division sentences by creating four linking number sentences. $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$
		Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences.	
Year 3/4	Division with a remainder	Divide objects between groups and see how much is left over. 13 ÷ 3 =	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.	Count times tables facts in head. Complete written divisions and show the
			-1 - 4 - 4 - 4 000000000000000000000000000000000000	$\begin{array}{c} 29 \div 8 = 3 \text{ REMAINDER 5} \\ \uparrow $
		14 ÷ 3 =	Draw dots and group them to divide an amount and	Abstract number line.
			clearly show a remainder.	



